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# Anisotropic discs loaded by parabolically distributed pressure

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**Abstract**

The complex potentials which describe the elastic equilibrium of a circular disc made of a transversely isotropic, homogeneous material are presented. It is assumed that the disc is loaded by a parabolic distribution of radial stresses which act along two finite circular arcs, antisymmetric with respect to the disc's center. The problem is here solved within the frame of linear elasticity assuming plane strain conditions. The complex potentials technique is adopted as it was formulated by Lekhnitskii in his pioneering contributions. When the complex potentials are determined, one can obtain the respective stress field developed all over the disc for any value of the angle between the axis of symmetry of pressure distribution and the planes of material isotropy. Attention is focused at the disc's center and at the loaded diameter. Conclusions regarding the applicability of the Brazilian-disc test in case of specimens made of transversely isotropic materials are drawn.



*Keywords:* Anisotropic materials; Transverse isotropy; Brazilian-disc test; Complex potentials, Stress tensor

## 1. Introduction

The Brazilian-disc test is the most convenient and widely used substitute of the direct tension test. It was initially introduced for isotropic materials (Carneiro 1943; Akazawa 1943; Hondros 1959). However, most rock-like materials exhibit some kind of anisotropy and therefore the applicability of existing solutions for stresses and displacements becomes questionable. The problem is usually confronted numerically (Dan & Konietzky 2014) or experimentally (Vervoort et al. 2014) since analytic solutions are very rare (Exadaktylos et al. 2001). In this context, an attempt to obtain such a solution is described here, using Lekhnitskii's (1968, 1981) complex potential technique for rectilinear

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anisotropic materials. The advantage of the solution introduced is that the disc is considered loaded by a parabolic (rather than uniform) distribution of radial stresses, an assumption much closer to reality Kourkoulis et al. (2012).

## 2. Theoretical considerations

Consider a linearly elastic, homogeneous and rectilinearly transversely isotropic body. Let E, ν and G be the Young's modulus, Poisson's ratio and shear modulus in the planes of isotropy, and E′, G′ the elasticity and shear moduli in planes normal to those of isotropy. Let also, ν′ be the Poisson's ratio defining the magnitude of dilatation in planes of isotropy for compression normally to them. Introducing a Cartesian reference system {O; x,y,z} with its origin at the disc's center and its y-axis normal to the planes of isotropy, the generalized Hooke's law reads as:

$$\begin{aligned}
\varepsilon_x &= \underbrace{\frac{1}{E}}_{\alpha_{11}}\sigma_x \underbrace{-\frac{\nu'}{E'}}_{\alpha_{12}}\sigma_y \underbrace{-\frac{\nu}{E}}_{\alpha_{13}}\sigma_z, \quad \gamma_{yz} = \underbrace{\frac{1}{G'}}_{\alpha_{44}}\tau_{yz}\\
\varepsilon_y &= \underbrace{-\frac{\nu'}{E'}}_{\alpha_{21}}\sigma_x + \underbrace{\frac{1}{E'}}_{\alpha_{22}}\sigma_y \underbrace{-\frac{\nu'}{E'}}_{\alpha_{23}}\sigma_z, \quad \gamma_{xz} = \underbrace{\frac{1}{G}}_{\alpha_{55}}\tau_{xz} = 2\frac{1-\nu}{E}\tau_{xz}\\
\varepsilon_z &= \underbrace{-\frac{\nu}{E}}_{\alpha_{31}}\sigma_x \underbrace{-\frac{\nu'}{E'}}_{\alpha_{32}}\sigma_y + \underbrace{\frac{1}{E}}_{\alpha_{33}}\sigma_z, \quad \gamma_{xy} = \underbrace{\frac{1}{G'}}_{\alpha_{66}}\tau_{xy}
\end{aligned} \tag{1}$$

It is seen that from the twelve non-zero strain coefficients $\alpha_{ij}$ only five ($\alpha_{11}$, $\alpha_{12}$, $\alpha_{13}$, $\alpha_{22}$ and $\alpha_{66}$) are linearly independent.

A cylindrical disc of radius R and thickness w is now cut from this body, with its cross-section normal to the planes of isotropy. This transversely isotropic disc, denoted from here on transtropic (for brevity reasons), is compressed, in complete absence of friction, between the jaws of the device suggested by the International Society for Rock Mechanics (ISRM) for the implementation of the Brazilian-disc test by an overall load $P_{frame}$ acting within its cross-section. $P_{frame}$ forms an arbitrary angle $\phi_o$ with respect to the material layers. Moreover, that $P_{frame}$ is parabolically distributed along two symmetric parts of the disc's lateral surface each one of area $2R\omega_o w$ (Fig. 1), following the law:

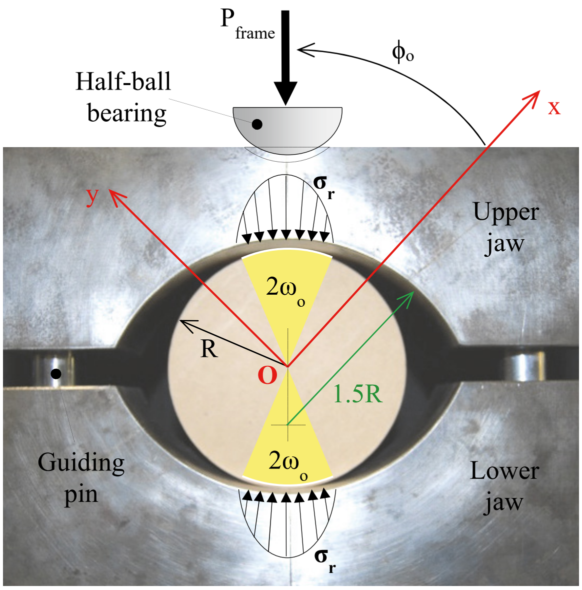


Fig. 1. The device suggested by the International Society for Rock Mechanics for the standardized implementation of the Brazilian-disc test.

$$\sigma_r = -P(\theta) = -P_c\left[1-\sin^2(\phi_o-\theta)/\sin^2\omega_o\right], \qquad \left(P_c = P(\theta)_{max},\ P(\theta)>0\right) \tag{2}$$

The elastic equilibrium of that transtropic disc is to be determined. In this context, the disc's cross-section is considered lying in the $z=x+iy=re^{i\theta}$ complex plane (Fig. 2). Assuming that w is comparable to R, plane strain conditions are adopted. Taking now into account Lekhnitskii's approach for anisotropic cylindrical bodies, with their faces being planes of elastic symmetry (Lekhnitskii 1981), the equilibrium equations, Hooke's generalized law, the equations of compatibility and the respectice boundary conditions, for zero body forces, are reduced respectively to:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0, \qquad \frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_x}{\partial y}=0 \tag{3}$$

$$\varepsilon_x=\frac{\partial u}{\partial x}=\beta_{11}\sigma_x+\beta_{12}\sigma_y, \quad \varepsilon_y=\frac{\partial v}{\partial y}=\beta_{12}\sigma_x+\beta_{22}\sigma_y, \quad \gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\beta_{66}\tau_{xy} \tag{4}$$

$$\frac{\partial^2}{\partial y^2}\left(\beta_{11}\sigma_x+\beta_{12}\sigma_y\right)+\frac{\partial^2}{\partial x^2}\left(\beta_{12}\sigma_x+\beta_{22}\sigma_y\right)-\frac{\partial^2}{\partial x\partial y}\beta_{66}\tau_{xy}=0 \tag{5}$$

$$\sigma_x\cos(n,x)+\tau_{xy}\cos(n,y)=X_n, \qquad \tau_{xy}\cos(n,x)+\sigma_y\cos(n,y)=Y_n \tag{6}$$

where u, v are the Cartesian components of displacement, $X_n$, $Y_n$ are the components of $\sigma_r$ on L and $\beta_{ij}=\alpha_{ij}-\alpha_{i3}\alpha_{j3}/\alpha_{33}$ are the so-called reduced elastic constants, which from Eqs.(1) equal:

$$\beta_{11}=\frac{1-\nu^2}{E}, \quad \beta_{12}=-\nu'\frac{1+\nu}{E'}, \quad \beta_{22}=\frac{1}{E'}\left(1-\frac{E}{E'}\nu'^2\right), \quad \beta_{66}=\frac{1}{G'} \tag{7}$$

According to the theory of elasticity, Eqs.(3) imply the existence of the Airy's function F(x,y) so that:

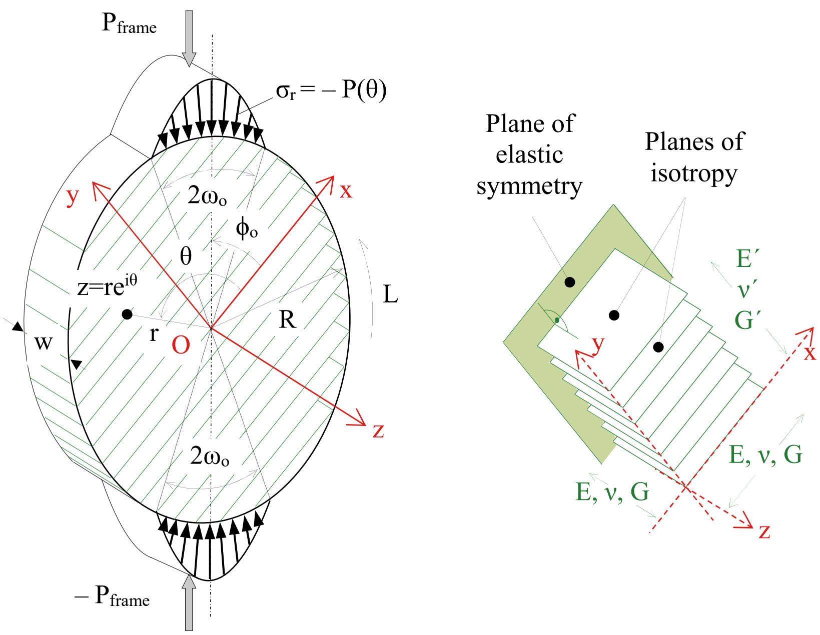


Fig. 2. The isolated transtropic disc under parabolic pressure: Configuration of the problem and definition of symbols.

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \tag{8}$$

Eq.(5) provides the generalized biharmonic equation in F:

$$\beta_{22}\frac{\partial^4 F}{\partial x^4} + \left(2\beta_{12} + \beta_{66}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2} + \beta_{11}\frac{\partial^4 F}{\partial y^4} = 0 \tag{9}$$

with a characteristic equation:

$$\beta_{11}\mu^4 + \left(2\beta_{12} + \beta_{66}\right)\mu^2 + \beta_{22} = 0 \tag{10}$$

Eq.(10) has four purely imaginary roots, the so-called complex parameters, as follows:

$$\mu_1 = i\beta_1, \quad \mu_2 = i\beta_2, \quad \mu_3 = \overline{\mu}_1 = -i\beta_1, \quad \mu_4 = \overline{\mu}_2 = -i\beta_2; \tag{11}$$

$$\left.\begin{matrix}\beta_1 \\ \beta_2\end{matrix}\right\} = \sqrt{\left|-\left(2\beta_{12} + \beta_{66}\right) \mp \sqrt{\left(2\beta_{12} + \beta_{66}\right)^2 - 4\beta_{11}\beta_{22}}\,\right| \Big/ \left(2\beta_{11}\right)} \in \mathbb{R}, \quad \beta_1 > \beta_2; \tag{12}$$

over-bar denotes the complex conjugate. Then, following Lekhnitskii (1981) it can be written:

$$F(x, y) = 2\Re\left[F_{11}(z_1) + F_{12}(z_2)\right] \tag{13}$$

with $z_1 = x + \mu_1 y$ and $z_2 = x + \mu_2 y$ the so-called complicated complex variables, and:

$$\Phi_1(z_1) = F_{11}'(z_1), \quad \Phi_2(z_2) = F_{12}'(z_2) \tag{14}$$

the Lekhnitskii's complex potentials (where prime denotes the first derivative).

$\Phi_1$ and $\Phi_2$ are obtained from the given values of stresses on the disc's boundary. Namely, stresses (Eqs.(8)) and their values on L (Eqs.(6)), expressed in terms of $\Phi_1$ and $\Phi_2$ as (Lekhnitskii 1981):

$$\sigma_x = 2\Re\left[\mu_1^2\Phi_1'(z_1) + \mu_2^2\Phi_2'(z_2)\right], \quad \sigma_y = 2\Re\left[\Phi_1' + \Phi_2'\right], \quad \tau_{xy} = -2\Re\left[\mu_1\Phi_1' + \mu_2\Phi_2'\right] \tag{15}$$

$$2\Re\left[\Phi_1(z_1) + \Phi_2(z_2)\right] = -\int_0^S Y_n dS, \quad 2\Re\left[\mu_1\Phi_1(z_1) + \mu_2\Phi_2(z_2)\right] = \int_0^S X_n dS \tag{16}$$

with S the arc length on L. According to Lekhnitskii (1968), $\Phi_1$ and $\Phi_2$ are sought in series form as:

$$\begin{aligned}\Phi_1(z_1) &= A_0 + A_1 z_1 + \sum_{n=2}^{\infty} A_n P_{1n}(z_1), \\ \Phi_2(z_2) &= B_0 + B_1 z_2 + \sum_{n=2}^{\infty} B_n P_{2n}(z_2);\end{aligned} \tag{17}$$

$$P_{1n,2n}(z_{1,2}) = -\left[R\left(1 - i\mu_{1,2}\right)\right]^{-n}\left\{\left[z_{1,2} + \sqrt{z_{1,2}^2 - R^2\left(1 + \mu_{1,2}^2\right)}\right]^n + \left[z_{1,2} - \sqrt{z_{1,2}^2 - R^2\left(1 + \mu_{1,2}^2\right)}\right]^n\right\} \tag{18}$$

When $z=re^{i\theta}$ is on L (r=R), then $z_{1,2}=R[(1-i\mu_{1,2})+(1+i\mu_{1,2})s^{-1}]/2$ with $s=e^{i\theta}$. Substituting in Eqs.(16) from Eqs.(17), (18), expanding the right-hand sides of Eqs.(16) in Fourier series form and comparing coefficients of the same order of $s=e^{i\theta}$, the following systems of equations are obtained for determining the real ($\Re$) and imaginary ($\Im$) parts of coefficients of $\Phi_1$ and $\Phi_2$ of Eqs.(17):

$$\left.\begin{matrix}\Re A_n\\ \Re B_n\end{matrix}\right\}=\frac{\left.\begin{matrix}-\beta_2\left(1-t_2^n\right)\\ +\beta_1\left(1-t_1^n\right)\end{matrix}\right\}\cdot\Re a_n+\left.\begin{matrix}+\left(1+t_2^n\right)\\ -\left(1+t_1^n\right)\end{matrix}\right\}\cdot\Im b_n}{\beta_2\left(1+t_1^n\right)\left(1-t_2^n\right)-\beta_1\left(1-t_1^n\right)\left(1+t_2^n\right)}$$
$$\left.\begin{matrix}\Im A_n\\ \Im B_n\end{matrix}\right\}=\frac{\left.\begin{matrix}-\beta_2\left(1+t_2^n\right)\\ +\beta_1\left(1+t_1^n\right)\end{matrix}\right\}\cdot\Im a_n+\left.\begin{matrix}-\left(1-t_2^n\right)\\ +\left(1-t_1^n\right)\end{matrix}\right\}\cdot\Re b_n}{\beta_2\left(1-t_1^n\right)\left(1+t_2^n\right)-\beta_1\left(1+t_1^n\right)\left(1-t_2^n\right)},\quad\left(n=3,5,...\right) \tag{19}$$

with $t_{1,2}=(1+i\mu_{1,2})/(1-i\mu_{1,2})$ and:

$$\left.\begin{matrix}a_3\\ b_3\end{matrix}\right\}=\begin{pmatrix}-1\\ +i\end{pmatrix}\frac{P_cR}{6\pi}\left[\sin 2\omega_o-\frac{1}{2\sin^2\omega_o}\left(\sin 2\omega_o-\omega_o-\frac{\sin 2\omega_o\cos 2\omega_o}{2}\right)\mp\frac{\sin 4\omega_o}{4}e^{-i2\phi_o}\right.$$
$$\left.\pm\frac{e^{-i2\phi_o}}{2\sin^2\omega_o}\left(\frac{\sin 4\omega_o}{2}-\frac{2\cos 2\omega_o\sin 4\omega_o-\sin 2\omega_o\cos 4\omega_o}{3}\right)\right]e^{-i2\phi_o},\quad\left(n=3\right) \tag{20}$$

$$\left.\begin{matrix}a_n\\ b_n\end{matrix}\right\}=\begin{pmatrix}-1\\ +i\end{pmatrix}\frac{P_cR}{2\pi n}\left\{\frac{\sin\left(n-1\right)\omega_o}{n-1}-\frac{1}{2\sin^2\omega_o}\left[\frac{\sin\left(n-1\right)\omega_o}{n-1}\right.\right.$$
$$\left.\left.+\frac{\left(n-1\right)\cos 2\omega_o\sin\left(n-1\right)\omega_o-2\sin 2\omega_o\cos\left(n-1\right)\omega_o}{4-\left(n-1\right)^2}\right]\right\}\left[e^{-i\left(n-1\right)\phi_o}+e^{-i\left(n-1\right)\left(\phi_o+\pi\right)}\right]$$
$$\begin{pmatrix}+1\\ +i\end{pmatrix}\frac{P_cR}{2\pi n}\left\{\frac{\sin\left(n+1\right)\omega_o}{n+1}-\frac{1}{2\sin^2\omega_o}\left[\frac{\sin\left(n+1\right)\omega_o}{n+1}\right.\right.\qquad\left(n=5,7,...\right)$$
$$\left.\left.+\frac{\left(n+1\right)\cos 2\omega_o\sin\left(n+1\right)\omega_o-2\sin 2\omega_o\cos\left(n+1\right)\omega_o}{4-\left(n+1\right)^2}\right]\right\}\left[e^{-i\left(n+1\right)\phi_o}+e^{-i\left(n+1\right)\left(\phi_o+\pi\right)}\right] \tag{21}$$

the Fourier series coefficients of the parabolic pressure induced on the disc ($a_n=b_n=0$, n=0,2,4,…). For n=1, one obtains the following relations (Lekhnitskii 1968):

$$\begin{aligned}&A_1+B_1+\bar{A}_1+\bar{B}_1=\left(a_1+\bar{a}_1\right)/R\\ &A_1\mu_1+B_1\mu_2+\bar{A}_1\bar{\mu}_1+\bar{B}_1\bar{\mu}_2=\left(\bar{a}_1-a_1\right)/\left(Ri\right)=\left(b_1+\bar{b}_1\right)/R\\ &A_1\mu_1^2+B_1\mu_2^2+\bar{A}_1\bar{\mu}_1^2+\bar{B}_1\bar{\mu}_2^2=\left(\bar{b}_1-b_1\right)/\left(Ri\right)\end{aligned} \tag{22}$$

with the respective Fourier series coefficients of parabolic pressure given as:

$$\left.\begin{matrix}a_1\\ b_1\end{matrix}\right\}=\begin{pmatrix}-1\\ +i\end{pmatrix}\frac{P_cR}{2\pi}\left[2\omega_o-\frac{2\omega_o-\sin 2\omega_o}{2\sin^2\omega_o}\mp\sin 2\omega_o e^{-i2\phi_o}\pm\frac{e^{-i2\phi_o}}{2\sin^2\omega_o}\left(\sin 2\omega_o-\omega_o-\frac{\sin 2\omega_o\cos 2\omega_o}{2}\right)\right] \tag{23}$$

Clearly, the three Eqs.(22) only provide the four terms $A_1$, $B_1$, $\overline{A}_1$ and $\overline{B}_1$ apart from an arbitrary real constant; for n=0, $A_0$ and $B_0$ remain completely arbitrary. Thus, the complex potentials $\Phi_1$, $\Phi_2$ characterizing the equilibrium of the elastic transtropic disc have been determined (with the only exception of a real and two complex constants) and therefore the problem should be considered solved, at least concerning the components of the stress field.

Regarding parameters $P_c$ and $\omega_o$, appearing in the above formulae, they can be arbitrarily predefined assuming that $P_{frame}$ remains constant. However, an alternative approach is proposed here, in order to achieve a more accurate approximation of the actual values of these quantities. In this direction, the formulae introduced by Markides and Kourkoulis (2012) for the respective disc-jaw contact problem for isotropic materials (Kourkoulis et al. 2012), are here further developed, in order to account also for a transtropic disc. It is then concluded that:

$$P_c(\phi_o)=\sqrt{\frac{3\pi P_{frame}}{32K(\phi_o)Rw}},\ \ \omega_o(\phi_o)=\operatorname{Arc}\sin\sqrt{\frac{6K(\phi_o)P_{frame}}{\pi Rw}},\ \ K(\phi_o)=\frac{\kappa(\phi_o)+1}{4G'}+\frac{\kappa_J+1}{4G_J} \tag{24}$$

with $\kappa(\phi_o)$, $\kappa_J$ and $G'$, $G_J$ the Muskhelishvili's (1968) constants and shear moduli of the disc's and jaw's cross-sections, respectively, as if both of them were made of isotropic materials. Moreover, for the plane strain conditions considered here and assuming that $\kappa(\phi_o)=3-4\nu(\phi_o)$, it can be seen that:

$$\nu(\phi_o)=\sqrt{\left(\frac{E}{E'}\nu'\cos\phi_o\right)^2+\left(\nu'\sin\phi_o\right)^2}=\nu'\sqrt{\frac{E^2}{E'^2}\cos^2\phi_o+\sin^2\phi_o} \tag{25}$$

### 3. Results and Discussion

Introducing $\Phi_1$ and $\Phi_2$ obtained before in the general formulae of Eqs.(15), the stress-field components can be calculated at any point of the transtropic disc. As an example, a disc of radius R=0.05 m and thickness w=0.01 m is considered here. The disc is made of a transtropic serpentinous schist. Its mechanical properties were provided by Barla and Innaurato (1973) and read as: E=58 GPa, E′=27 GPa, ν=0.34 and ν′=0.12. The disc is compressed by an overall force equal to $P_{frame}$=20 kN. Fot the numerical calculations it was considered that n=59 for the additional terms in all previous formulae. Finally, $P_c$ and $\omega_o$ are calculated from Eqs.(24) and (25) whereas G′ is determined using the following formulae (Lekhnitskii 1981):

$$G'\simeq\frac{EE'}{E(1+2\nu')+E'} \tag{26}$$

As a first step, attention is focused at the disc's center, given that the stress tensor at this point is crucial for the sound determination of the tensile strength according to the initial concept of Carneiro (1943) and Akazawa (1943). The ratio, $\xi$, of the normal stresses, i.e. the ratio $\xi=\sigma_r/\sigma_\theta$ of the radial- over the transverse-stress at the specific point is plotted in Fig.3. It is clear from this figure that $\xi$ strongly differs from the respective value for the isotropic disc, which is equal to 3 (Hondros 1959), almost independently of the actual stress distribution along the loaded rim (Fairhurst 1964; Hobbs 1965; Hudson et al. 1972; Kourkoulis et al. 2013; Markides & Kourkoulis 2012). For the specific combination of mechanical properties, the value of $\xi$ varies from a maximum value equal to about 5.0 for $\phi_o=0^o$ to a value equal to about 2.2 for $\phi_o=90^o$.

Equally important is the fact that the shear stress component $\tau_{r\theta}$ at the disc's center is non-zero, in spite of the geometry and loading symmetry. Although the magnitude of these stresses is a rather small portion of the respective transverse stress $\sigma_\theta$, it is by no means ignorable for the whole range of for $\phi_o$ angles. The variation of the $\tau_{r\theta}/\sigma_\theta$ ratio versus the angle $\phi_o$ is plotted in Fig.4. It is observed that this variation is not monotonous. A clear extremum appears for $\phi_o=30^o$. This non-monotonous behavior should be expected, since for $\phi_o=0^o$ and $\phi_o=90^o$ the shear stress must be zeroed due to the additional symmetry of the orientation of the material layers with respect to the loading axis. From a quantitative point of view, the shear stress attains values equal to almost one fourth ($\tau_{r\theta}/\sigma_\theta=0.238$) of the respective transverse stress at $\phi_o=30^o$.

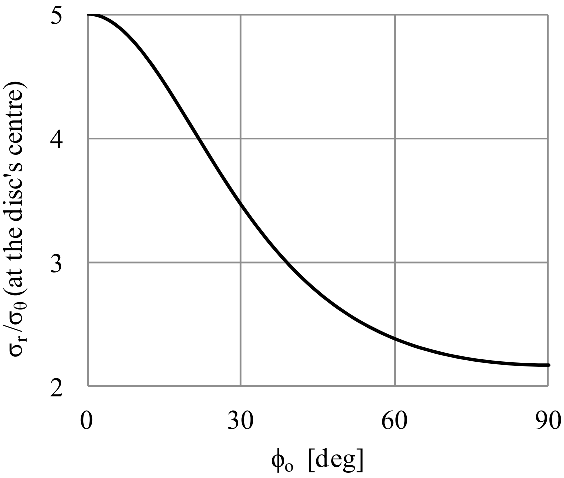


Fig. 3. The radial- over the transverse-stress at the disc's center against angle $\phi_o$.

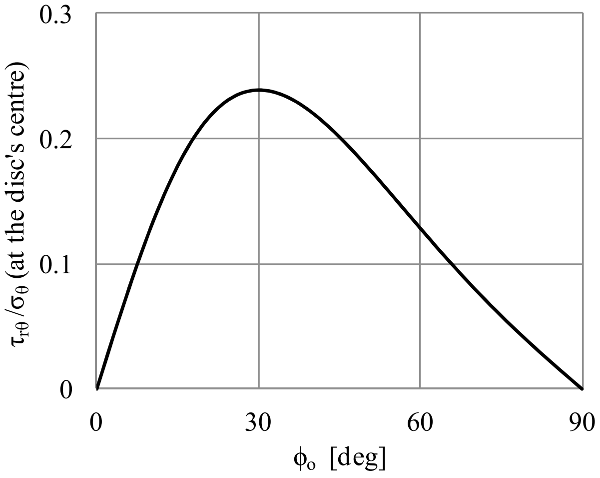


Fig. 4. The shear- over the transverse-stress at the disc's center against angle $\phi_o$.

Both points discussed in the previous two paragraphs raise serious questions concerning the applicability of the Brazilian-disc test for the determination of the tensile strength of transtropic materials. Indeed, it has been definitely clarified by many researchers (see for example Jaeger (1967) and Fairhurst (1964)), that in order for the quantity determined by the Brazilian-disc test to be a reasonable representation of the material's tensile strength, fracture must start from the disc's center and for this to be guaranteed $\xi$ should lie within specific limits.

As a next step the variation of the stress components all along the loaded diameter (i.e. along the symmetry axis of the parabolic distribution of the radial stresses acting along the loaded rims) is considered. Adopting the same as above numerical values for the problem parameters, the stress components are calculated and plotted from r=0 to r=R in Fig.5. Three cases are considered for angle $\phi_o$ equal to 15$^o$, 45$^o$ and 75$^o$. In the embedded figure a magnified view of the distribution of the stress components around the disc's center is shown. It is seen that from a quantitative point of view (and besides the presence of shear stresses) the distributions of the normal stresses closely resemble

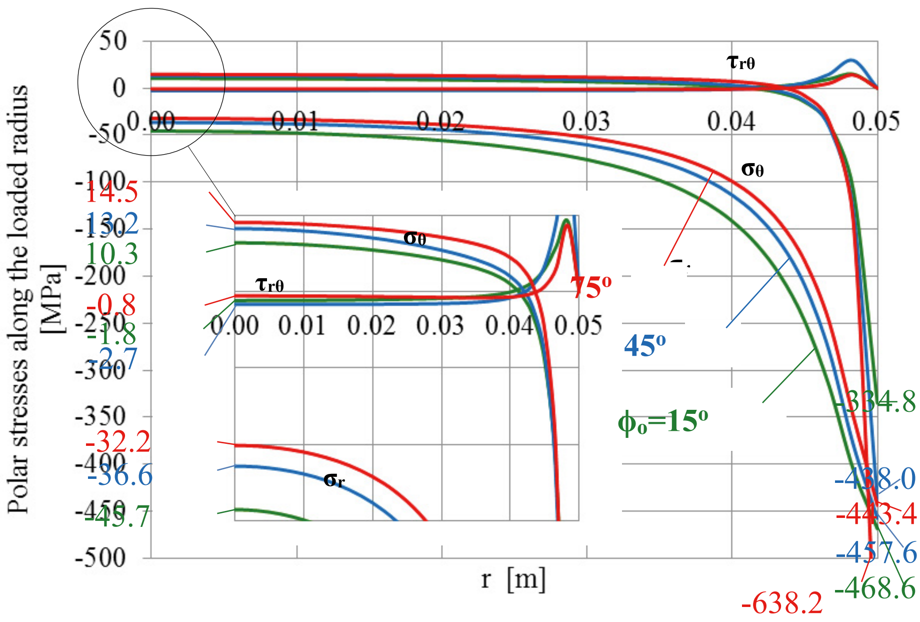


Fig.5. The distribution of the normal and shear stresses along the loaded diameter for three characteristic values of the angle $\phi_o$. The embedded figure is a magnified view of the stress distribution relatively close to the disc's center.

the respective ones of the isotropic disc. The main difference is the fact that for r=R, i.e. on the disc's periphery, the normal stresses (radial and transverse) are not equal to each other, i.e. $\sigma_\theta \neq \sigma_r$, while in the case of an isotropic disc, for r=R, it holds that $\sigma_\theta = \sigma_r$ (Markides and Kourkoulis 2012). Equally important is the fact that as one approaches the loaded rim (i.e. for r→R), the shear stress component starts increasing abruptly before it becomes zero for r=R.

## 4. Conclusions

The complex potentials characterizing the equilibrium of an elastic circular disc made of a transversely isotropic material were determined analytically. The disc was considered under the action of a parabolic distribution of radial stresses along two antisymmetric arcs of its periphery, which closely resembles the pressure induced on the disc in case it is squeezed between two curved metallic jaws. Moreover, the length of the loaded arcs was assumed to be a function of the load induced and the mechanical propreties of the materials of both the disc and the jaws.

Taking advantage of the complex potentials, it was possible to explore the stress field at the disc's center and also all along the loaded diameter. It was concluded that the applicability of the Brazilian-disc test becomes questionable for two reasons: The ratio of the transverse-to-normal stress at the disc's center is not constant and also shear stresses appear, which for specific values of the angle between the loaded diameter and the material layers may even reach one fourth of the respective transverse stress. As a result, it cannot be a-priori guaranteed that facture starts from the disc's center (a requirement sine-qua-non for the test to provide reasonable results). It is thus strongly suggested to avoid using the Brazilian-disc test for the determination of the tensile strength of transtropic materials unless a proper fracture criterion is first applied.

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